Explained Variation and Discrimination Measures for Censored Survival Data

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The assessment of the predictive performance of a regression model relating a survival time outcome to a set of covariates, is a vital step to evaluate if a model is suitable to be used for predictive purposes.
Among the measures proposed in the biostatistical literature, those based on discrimination and explained variation evaluate key aspects of the model performance. The former class refers to the model ability to segregate groups of patients according to the actual outcome. Discrimination measures are rank based indicators which consider the consistency between the rank of observed outcomes and the rank of model predictions, for pairs of patients. Explained variation measures refer to the extent to which the model determine individual outcome and consider the consistency between the model predicted and actual outcome at the level of the single patient, assessed via a loss (inaccuracy) function. Explained variation measures are derived by comparing the reduction of inaccuracy obtained when a regression model is used instead of a null model.

The biostatistical literature concerning discrimination and explained variation measures is fragmented and difficult to rejoin to a common class of measures. Moreover within each class of measures different estimation techniques are presented.

In the present work, a reinterpretation of the commonly used indicators of model discrimination, i.e. area under the ROC curve (AUC) and Harrell's C index, is provided. These are seen as measures of 'explained discrimination' based on a new concept of loss function functions, defined, for each patient, by comparing: the rank of his/her actual outcome with any other patient, and the rank between the corresponding predictions. Concerning explained variation, the literature shows: measures obtained from inaccuracy functions based on the comparison between predicted and observed survival time (Korn and Simon, 1990), and measures derived as a weighted average over time of the explained variation of the prediction at a given time point, as in the works of Graf et al (1999) and Schemper (2003). Following the approaches reviewed for explained variation, a weighted average over time of AUCs is presented as a new summary measure of discrimination. An estimator is derived addressing censoring via inverse-probability-of-censoring-weighting.

References:

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